under which we should use the expressions (10) or (16) instead of (1).

We shall begin with the analysis of the relation between the solutions (10) and (16).

For illustration let us calculate the characteristic percentage differences which arise by approximation of (13) by means of (15):

for $B = -0.1$ one finds a difference of 2.7%,

for $B = -0.2$ one finds a difference of 9.9%.

In many operating regimes of convective dryers $|-B| < 0.2$. In such conditions approximation of the solution (10) can be allowed by means of equation (16).

Comparing equations (1) and (16) we can conclude that

$$\theta = \theta^0,$$

if we accept

$$k_Y = k_Y^0 \beta. \tag{17}$$

On the basis of (17) we can conclude that in the region mentioned (that is the regime wherein most of the commercial convective dryers are operating) the mass-transfer coefficient k_Y can be considered to be independent of the humidity potential, but it is strongly dependent on saturation humidity level (or wet-bulb temperature). This conclusion has been fully confirmed by results of the comparison of experimental data and some theoretical solutions performed by Bobe *et al.* [8]. The conclusion quoted is in accordance with the theoretical analysis of Ashworth and Keey [9]. It is interesting to note that the dependence expressed by a very simple relation (17) is quantitatively in good accordance with their numerical results. This comparison is shown in Fig. 2.

As we have seen, the increase of absolute values of the parameter $|B|$ causes continuous enlargement of the difference between the solutions (10) and (16). For illustration, let us give some values of percentage difference between (10) and (16) for the parameter values $|B| > 0.2$:

for $B = -0.25$ one finds a difference of 17.2%,

for $B = -0.3$ one finds a difference of 23.6% etc.

From this analysis we can conclude that for the operating regimes with the parameter values $|B| > 0.2$ the solution (10) or (12) should be used for determination of the drying time.

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PHOTOSENSITIZED REACTIONS IN ABSORBING–SCATTERING MEDIA WITHIN A PLANE SLAB*

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NOMENCLATURE

$\dot{a}'''$,	dimensionless local rate of energy absorption;
c,	scattering albedo;
$\dot{e}'''$,	dimensional local rate of energy absorption;
$\dot{\mathcal{E}}''$,	power absorbed within the slab per unit interfacial area;
E_1,	first order exponential integral;
I,	whole radiant energy incident in x;
I_0,	intensity of the incident radiation;
m,	$= (\beta+\sigma)s$ optical thickness;
s,	width of the slab;
x,	coordinate.

Greek symbols

β,	absorption coefficient;
θ,	angle between a generical direction and the x-axis;
μ,	$= \cos\theta$;
σ,	scattering coefficient.

ATTENTION has been recently paid to the role of scattering in photochemical reactions [1–3].

Coupled absorbing and scattering phenomena are difficult to be handled since the radiant energy balance equation is an integro-differential one even in the case of uniform composition with regard to the radiation process (photosensitized reaction) [3].

While a statistical approach based on Monte Carlo technique appears to be the best tool to handle situations of finite geometrical dimensions [3], analytical or semi-analytical solutions are, on the contrary, feasible for one-dimensional situations, mainly for plane slabs. These last geometries must be considered only as a limit configuration of the reaction vessel. They are nevertheless worth being investigated in order to confirm on a stronger computational basis the physically plausible results obtained following the Monte Carlo approach.

A slab of finite width s has been considered with a parallel plane monochromatic radiation of intensity I_0 which impinges normally on the plane $x = 0$, while the face $x = s$ has been assumed to be transparent. Only absorption and scattering have been considered within the medium since emission is usually negligible in photochemical processes: only the case of isotropic scattering has been further examined.

For the situation at hand the local rate of energy absorption is given by

$$\dot{e}'''(x) = \beta I(x) \tag{1}$$

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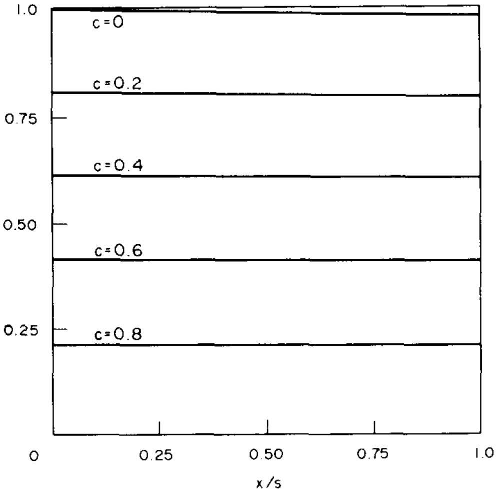

FIG. 1. $\dot{a}'''(x)$ distribution for $m = 0.02$.

where

$$I(x) = 2\pi \int_{-1}^{1} I(x,\mu)\,\mathrm{d}\mu \qquad (2)$$

is the whole radiant energy incident in x.

$I(x)$ can be obtained as the solution of the linear integral equation

$$I(x) = I_0 e^{-(\beta+\sigma)x} + \frac{\sigma}{2}\int_0^s E_1(|x-x'|)I(x')\,\mathrm{d}x' \qquad (3)$$

where β and σ are the absorption and scattering coefficients respectively.

Several methods have been proposed for the solution of equation (3) [4]: a method based on the Fourier integral transform has been used here. Such a method proposed by Boffi [5] makes itself recommended for its high reliability and for its simple computational requirements, once its mathematics has been performed. Details on the method and the solution procedure can be found in [6, 7]. Results used here have been obtained for an order of approximation $N = 10$.

Once $I(x)$ has been obtained, if $\dot{e}_0''' = (\beta+\sigma)I_0$ is assumed as the characteristic scale for the local rate of energy absorption, results can be given in dimensionless form as

$$\dot{a}'''(x) = (1-c)\frac{I(x)}{I_0} \qquad (4)$$

where $c = \sigma/\beta+\sigma$ is the scattering albedo.

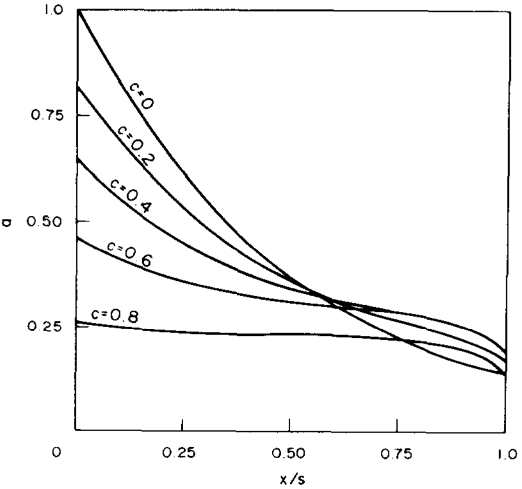

FIG. 2. $\dot{a}'''(x)$ distribution for $m = 2.0$.

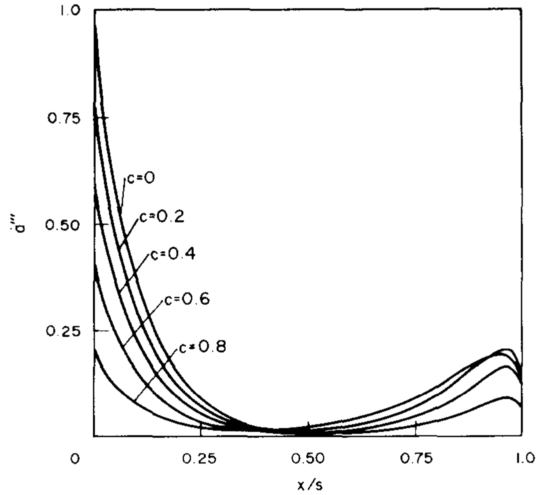

FIG. 3. $\dot{a}'''(x)$ distribution for $m = 10.0$.

Once the $\dot{a}'''(x)$ distribution has been evaluated, the power absorbed within the slab volume per unit interfacial area given as

$$\dot{\mathscr{E}}'' = \int_0^s \dot{a}'''(x)\,\mathrm{d}x \qquad (5)$$

can be easily obtained.

Both the quantities (4) and (5) are relevant in the analysis of photosensitized processes depending on the mixing state of the reacting system.

Results for different values of m (the optical thickness based on the total extinction coefficient) and c are given in Figs. 1–3.

The effect of scattering can be inferred by comparison of the results for the actual values of m and c with those for the same m and absorption only ($c = 0$).

The influence of scattering on the $\dot{a}'''$ distribution is strongly dependent on the value of m. At low values of m the mean free path of a photon is large when compared with the significant geometric dimension of the system and almost all the scattered photons escape through the boundaries of the slab. Therefore, since c is the probability for scattering to occur, for an optically thin situation ($m = 0.02$), the $\dot{a}'''$ distribution varies with c in an extent which is roughly proportional to $(1-c)$ (Fig. 1).

When the optical thickness increases, an increasing fraction of the scattered photons doesn't succeed to escape from the system but will experience absorption within the slab in such a way to reduce the non-uniformity of the exponentially decreasing $\dot{a}'''$ distribution of the pure absorption case (Figs. 2, 3).

Even in these situations, however, scattering causes a reduction of the power absorbed within the slab (with respect to the pure absorption situation) but the loss is, of course, lower the higher the value of m is as it is apparent from the results summarized in Table 1.

On the basis of the results for the absorption of radiant energy, some qualitative conclusions can be drawn for the

Table 1. Ratio of the actual absorbed power to the power absorbed in pure absorption situation with the same total extinction coefficient

	m		
c	0.02	2.0	10.0
0.	1.0	1.0	1.0
0.2	0.8078	0.9424	0.9646
0.4	0.6118	0.8616	0.9155
0.6	0.4120	0.7377	0.8442
0.8	0.2081	0.5182	0.7135

influence of scattering on the conversion which can be achieved in a photosensitized reaction occurring in a fluid flowing within the slab.

The mixing state of the reacting fluid is the major point in extending the results of the previous analysis to the chemical reaction since it has been shown [8, 9] that, in the two extreme situations of perfect mixing in the plane normal to the flow direction and of no mixing, conversion depends on the totally absorbed energy (no matter which the $\dot{a}'''$ distribution is) and on the $\dot{a}'''$ distribution respectively.

It can be therefore concluded that scattering will reduce conversion in all those situations where a good radial mixing occurs and therefore conversion is more sensitive to $\dot{\mathcal{E}}''$ than to the $\dot{a}'''$ distribution.

On the contrary scattering can affect positively conversion when, for high values of m, poor radial mixing conditions are considered. In these optical situations, for these flow conditions, the rearrangement of the $\dot{a}'''$ distribution causes the absorbed energy to be more effective for the reaction: this positive effect may overcome the negative effect due to the reduction of $\dot{\mathcal{E}}''$ so that an increase of conversion with respect to the pure absorption situation is the final result. In optically thick media scattering acts therefore as a mixing promoter of the absorbed energy and, in the absence of a satisfactory fluidinamic mixing, an increase of conversion follows consequently from the improvement of the total degree of mixing of the reacting system. Even in these situations, of course, conversion will be improved up to a value of c, depending on m, since in the limit of pure scattering medium ($c \to 1$) no absorption occurs within the slab and no reaction can occur.

As a concluding remark it is worth noting that these results confirm conclusions drawn on the effect of scattering on photosensitized reactions in an annular reactor of finite dimensions [3].

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A SIMPLE ANALYSIS OF UNSTEADY HEAT TRANSFER IN IMPULSIVE FALKNER–SKAN FLOWS

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NOMENCLATURE

a,	relaxation parameter defined by equation (19);
f,	dimensionless velocity component parallel to the wall {see equation (9a) of [1]};
$f''(0)$,	second derivative of f with respect to η evaluated at the wall;
$f_s''(0)$,	steady state value of $f''(0)$;
G_n,	functions of ξ defined by equation (6b);
H_n,	functions of ξ defined by equation (6c);
n,	exponent of the coordinate parallel to the wall in inviscid flow relation;
Pr,	Prandtl number;
V,	dimensionless normal velocity component {see equation (9c) of [1]};
V_n,	functions of ξ defined by equation (6a).

Greek symbols

β,	wedge angle equal to $2n/(n+1)$;
δ,	parameter given as $(1-n)/(1+n)$;
ξ,	independent variable defined by equation (5);
η,	transformed quasi-similar coordinate {see equation (8b) of [1]};
θ,	dimensionless temperature;
ϕ_0,	steady state value of $-\partial\theta/\partial\eta$ evaluated at the wall;
τ,	dimensionless time {see equations (8a) and (14) of [1]}.

Superscripts

$'$ $''$,	to denote first and second derivatives with respect to ξ or η.

INTRODUCTION

IN A RECENT paper, Watkins [1], presented the numerical solution for predicting unsteady heat-transfer coefficient in the case of impulsive Falkner–Skan laminar flows. As clearly explained in that work numerical estimates are not easy to obtain due to the characteristic curve dividing, at all times, Rayleigh and Falkner–Skan boundary-layer regions which must be matched through a transition zone.

The purpose of this short contribution is to show that previous approximate methods developed to predict the rate of heat transfer at unsteady state conditions, but when the flow was steady, (Sparrow and Gregg [2], Cess and Sparrow [3] and Cess [4]) are also of direct application to the present situation.

In a recent analysis [5] the authors have shown that in the case of small Prandtl (Pr) numbers, in the above mentioned works, the convective effect was in actual facts overestimated since the velocity profile was approximated by the leading term of an expansion series from the wall.

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